**UNIVERSITY OF TORONTO  
Faculty of Arts and Science**

**APRIL 2016 EXAMINATIONS**

**PHL245H1-S**

**Alex Koo**

**Duration - 3 hours**

**Examination Aid: Sheet with rules (provided)**

Last Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

First Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Student Number: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Answer **ALL** questions on the exam paper.

Use examination booklets for rough work if needed.

If you need further space, use an examination booklet and clearly indicate on the exam paper where your solution is.

The exam consists of 16 pages. Pages 2-14 have questions on them.

The final two pages (15-16) are an aid sheet and may be detached from the rest of the exam.

Part I: Semantics (30 marks)

1. Provide a shortened truth-table that demonstrates the following set of sentences is consistent. (3)

{Q↔R∧T, ~(P∨Q), P→~(R→S)}

2. Suppose we have an argument where the set of premises is inconsistent. What, if anything, can we conclude about the validity of the argument? Briefly explain your answer. (3)

3. What does it mean for a sentence in predicate logic to be a logical truth/tautology? If a sentence that is a logical truth/tautology is the conclusion to an argument, what can we conclude about the argument? Briefly explain your answer. (3)

4. Provide an English explanation that shows the following sentence is a contradiction. (4)

(∃x~Fx→~∀yGy)↔∀zGz∧~∀yFy

5. Provide an intensional (English language) interpretation to demonstrate that this argument is invalid. (3)

∃x(Fx∧~Gx). ∃xGx. ∃y∀x(Gx→(Fy∧L(yx)). ∴ ∀x~L(xx).

6. Provide an extensional interpretation (finite abstract model) of the following set of sentences that shows them to be consistent. (3)

Fa. ∀x(Gx→∃z~Fz). ∀y(Hy↔~Gy). ~Ha.

7. Provide an extensional interpretation (finite abstract model) of the following argument that shows it to be invalid. (3)

∀x~(Fx↔Ax). ∃x(Ax∧~(L(xx)∨Gx)). ∀x(Fx→∃y(Gy∧L(yx))). ∴ ∀xAx.

8. A restriction on Universal Instantiation is that the letter you instantiate to cannot occur as a bound variable within the original sentence. Provide an example of a universal instantiation that **violates** this restriction. Explain why the restriction makes logical sense. (3)

9. Consider the following argument:

∀x∃y(G(xy)→~G(yx)). ∃y(∀xG(yx)∧Ay). ∴ ~∀x(Ax→L(xx)).

a. Provide a truth-functional expansion to a universe of discourse with 2 elements. (4)

b. Provide an extensional interpretation (finite abstract model) that shows the argument to be invalid. (1)

Part II: Symbolization (34 Marks)

Symbolize questions 1-9, and translate question 10 using the provided abbreviation schemes.

1. Alfred, who is Bruce Wayne’s butler, is nice to everyone except for Superman. (3)

F1: *a* is a person. G2: *a* is nice to *b*. a0: Alfred. b0: Bruce Wayne. c0: Superman.

b1: The butler of *a*.

2. Some dogs don’t chew shoes. (2)

D1: *a* is a dog. G1: *a* is a shoe. C2: *a* chews *b*.

3. Although a bear is a scary animal, Joe’s cousin is not scared of any bear. (3)

A1: *a* is an animal. B1: *a* is a bear. D1: *a* is scary. D2: *a* is scared of *b*.   
 a0: Joe. a1: The cousin of *a*.

4. Tim’s cousin only buys toothbrushes from the store next to Walmart. (3)  
A1: *a* is a toothbrush. D1: *a* is a store. B3: *a* buys *b* from *c*. a0: Tim. b0: Walmart.   
a1: The cousin of *a*. b1: The store next to *a*.

5. Some coffee shop or another is visited by every student that doesn’t like tea. (4)

C1: *a* is a coffee shop. D1: *a* is a student. F1: *a* is a tea. G2: *a* likes *b*.   
K2: *a* visits *b*.

6. Even though there are at most one comedians that Bob doesn’t follow on Twitter, no one finds Bob funny. (4)

C1: *a* is a comedian. D1: *a* is a person. A2: *a* finds *b* funny.   
F3: *a* follows *b* on *c*. b0: Bob. d0: Twitter.

7. People ride the scariest rollercoaster at Canada’s Wonderland when, and only when they are not afraid of heights. (4)

F1: *a* is a person. H1: *a* is a height. a0: Canada’s Wonderland

A2: *a* is a rollercoaster at *b*. B2: *a* is afraid of *b*. C2: *a* is scarier than *b*. G2: *a* rides *b*.

8. Opera and ballet are boring unless they are performed by some professionals, and the former case is necessary for people to buy tickets to watch them. (4)

A1: *a* is an opera. B1: *a* is a ballet. C1: *a* is boring. D1: *a* is a professional.   
F1: *a* is a person. G1: *a* is a ticket. M2: *a* performs *b*. N3: *a* buys *b* for *c*.

9. Symbolize the following ambiguous sentence TWO logically distinct ways. For each, provide an English sentence that clearly explains the meaning of symbolization. (4)

Someone is always eating a cheesecake.

F1: *a* is a person. C1: *a* is cheesecake. D1: *a* is a time. E3: *a* is eating *b* at time *c*.

10. Translate the following symbolic sentence into an IDIOMATIC English sentence. (3)

∃x∀y(Fy∧∃w∃z(Dw∧Dz∧w≠z∧G(ywb(a))∧G(yz(b(a)))↔y=x))

F1: *a* is a person. D1: *a* is a leprechaun. G3: *a* saw *b* on *c*. a0: Friday. b1: The morning of *a*.

Part III: Derivations (36 marks)

1. Show the following argument is valid using a derivation. Use only the **basic rules**: MP, MT, ADD, MTP, ADJ, S, R, DN, CB, BC, UI, EI, and EG. (9)

∀y(∼∃zH(zy)→∼Fy). ∀y∀z∼(D(zy)∧H(yz)). ∀w∀z(G(a(az)w)∨D(wz)) ∴ ∀x(Fx→∃yG(yx))

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2. Show the following argument is valid using a derivation. Use only the **basic rules**: MP, MT, ADD, MTP, ADJ, S, R, DN, CB, BC, UI, EI, and EG. (9)

∀x∼(∼Fx↔∃yB(xy)). ∀x∃y∀zH(za(b(xy))). ∀x∃zB(xb(z)). ∴ ∀wFw∧∃xH(xa(x))

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3. Prove that the following statement is a theorem by constructing a derivation. You may use **all rules**. (9)

∴ ∀x(∀yB(xy)↔Ax)→∃x∀y(Ay→B(xy))

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4. Prove that the following argument is valid using a derivation. You may use **all rules**. (9)

∀x(∼Fx∨∀z∃wH(zw))→∀x∀yM(b(a(y))x). ∀z(Fz→∀w∃yH(wy)). ∴ ∃xM(b(x)x)

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Total = 100Marks

**AID SHEET: Derivation Rules (2 pages)**

**Derivation Types:**

**Direct Derivation (DD or dd)**

**Conditional Derivation (CD or cd)**

**Indirect Derivation (ID or id)**

**Universal Derivation (UD or ud)** Restriction: the instantiating term cannot occur  
 unbound in any available line, or in a premise  
 used in an available line.

**---------------------------------------------------------------------------------------------------------------------**

**10 Basic Rules for Sentential Operators:**

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| **Modus Ponens (MP or mp)**  (φ → ψ)  φ  ⎯⎯⎯⎯  ψ | | | | **Modus Tollens (MT or mt)**  (φ → ψ)  ~ψ  ⎯⎯⎯⎯  ~φ | |
| **Double Negation (DN or dn)** | | | | **Repetition (R or r)** | |
| φ  ⎯⎯⎯  ~~ φ | | ~~φ  ⎯⎯⎯  φ | | φ  ⎯⎯⎯  φ | |
| **Simplification (S/SL/SR or s/sl/sr)** | | | | **Adjunction (ADJ or adj)** | |
| φ ∧ ψ  ⎯⎯⎯⎯  φ | | | φ ∧ ψ  ⎯⎯⎯⎯  ψ | φ  ψ  ⎯⎯⎯⎯  φ ∧ ψ | |
| **Addition (ADD or add)** | | | | **Modus Tollendo Ponens (MTP or mtp)** | |
| φ  ⎯⎯⎯⎯  φ ∨ ψ | ψ  ⎯⎯⎯⎯  φ ∨ ψ | | | φ ∨ ψ  ~ φ  ⎯⎯⎯⎯  ψ | φ ∨ ψ  ~ ψ  ⎯⎯⎯⎯  φ |
| **Biconditional-Conditional (BC or bc)** | | | | **Conditional-Biconditional (CB or cb)** | |
| φ ↔ ψ  ⎯⎯⎯⎯  φ → ψ | | | φ ↔ ψ  ⎯⎯⎯⎯  ψ → φ | φ → ψ  ψ → φ  ⎯⎯⎯⎯  φ ↔ ψ | |

**Derived Rules for Sentential Operators:**

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| **Negation of Conditional**  **(NC or nc)** | ~(φ → ψ)  ⎯⎯⎯⎯  φ ∧ ~ψ | | | φ ∧ ~ψ  ⎯⎯⎯⎯  ~(φ → ψ) | | |
| **Conditional as Disjunction**  **(CDJ or cdj)** | φ → ψ  ⎯⎯⎯⎯  ~φ ∨ ψ | ~φ ∨ ψ  ⎯⎯⎯⎯  φ → ψ | | ~φ → ψ  ⎯⎯⎯⎯  φ ∨ ψ | φ ∨ ψ  ⎯⎯⎯⎯  ~φ → ψ | |
| **Separation of Cases**  **(SC or sc)** | φ ∨ ψ  φ → χ  ψ → χ  ⎯⎯⎯⎯  χ | | | φ → χ  ~φ → χ  ⎯⎯⎯⎯  χ | | |
| **De Morgan’s**  **(DM or dm)** | ~ (φ ∨ ψ)  ⎯⎯⎯⎯  ~φ ∧ ~ψ | | ~ (~φ ∨ ~ψ)  ⎯⎯⎯⎯⎯  φ ∧ ψ | ~ (φ ∧ ψ)  ⎯⎯⎯⎯  ~φ ∨ ~ψ | | ~ (~φ ∧ ~ψ)  ⎯⎯⎯⎯⎯  φ ∨ ψ |
| ~φ ∧ ~ψ  ⎯⎯⎯⎯  ~ (φ ∨ ψ) | | φ ∧ ψ  ⎯⎯⎯⎯⎯  ~ (~φ ∨ ~ψ) | ~φ ∨ ~ψ  ⎯⎯⎯⎯  ~ (φ ∧ ψ) | | φ ∨ ψ  ⎯⎯⎯⎯⎯  ~ (~φ ∧ ~ψ) |
| **Negation of Biconditional**  **(NB or nb)** | ~ (φ ↔ ψ)  ⎯⎯⎯⎯  φ ↔ ~ ψ | | | φ ↔ ~ψ  ⎯⎯⎯⎯  ~ (φ ↔ ψ) | | |

**Derivation Rules for Predicate Logic:**

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| **Basic Rule:**  **Existential**  **Generalization**  **(EG or eg)** | **Basic Rule:**  **Universal Instantiation (UI or ui)** | **Basic Rule:**  **Existential Instantiation  (EI or ei)** | **Derived Rule:**  **Quantifier Negation (QN or qn)** | | **Derived Rule:**  **Alphabetic Variance  (AV or av)** | |
| φζ  \_\_\_\_\_  ∃αφα  Restriction: α does not occur as a free or bound variable in φζ. | ∀αφα  \_\_\_\_\_  φζ  Restriction: ζ does not occur as a bound variable in φα | **∃**αφα  \_\_\_\_\_  φζ  Restriction: ζ does not occur in any previous line or premise. | ∀α~φ  \_\_\_\_\_ ~∃αφ | ~∀αφ  \_\_\_\_\_  ∃α~φ | ∀αφα  \_\_\_\_\_  ∀βφβ | ∃αφα  \_\_\_\_\_  ∃βφβ |
| ~∃αφ  \_\_\_\_\_  ∀~αφ | ∃α~φ  \_\_\_\_\_  ~∀αφ | Restriction: β does not occur as a bound variable in φα | |

Total Pages (16)